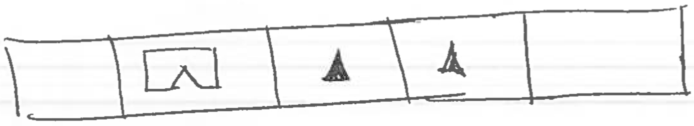
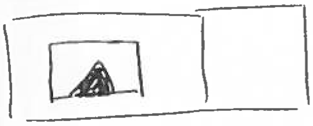
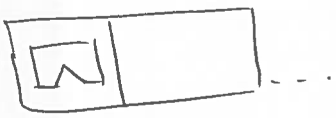
(1)

ligands ▲ , L

boxes □ , Ω

a)



STATE	MULTIPLICITY	ENERGY
Bound 	$\frac{\Omega!}{(L-1)!\,(\Omega-L+1)!}$	$\varepsilon_b + (L-1)\,\varepsilon_{sol}$
unbound 	$\frac{\Omega!}{L!\,(\Omega-L)!}$	$L\,\varepsilon_{sol}$

b)

$$P_{bound} = \frac{\frac{\Omega!}{(L-1)!\,(\Omega-L+1)!}\exp\left[-\beta\left(\varepsilon_b + (L-1)\,\varepsilon_{sol}\right)\right]}{\frac{\Omega!}{L!\,(\Omega-L)!}\exp\left[-\beta\left(L\varepsilon_{sol}\right)\right] + \frac{\Omega!}{(L-1)!\,(\Omega-L+1)!}\exp\left[-\beta\left(\varepsilon_b + (L-1)\varepsilon_{sol}\right)\right]}$$

c) using $\frac{\Omega!}{(\Omega-L)!} \approx \Omega^L$ simplify (b):

$$P_{bound} = \frac{\frac{\Omega^{L-1}}{(L-1)!} \exp\{-\beta[\varepsilon_b + (L-1)\varepsilon_{sol}]\}}{\frac{\Omega^L}{L!} \exp(-\beta L\varepsilon_{sol}) + \frac{\Omega^{L-1}}{(L-1)!} \exp\{-\beta[\varepsilon_b + (L-1)\varepsilon_{sol}]\}}$$

$$= \frac{1}{\frac{\Omega^L}{L!} \frac{(L-1)!}{\Omega^{L-1}} \exp(-\beta L\varepsilon_{sol}) \exp(+\beta[\varepsilon_b + (L-1)\varepsilon_{sol}]\}) + 1}$$

$$= \frac{1}{\frac{\Omega}{L} \exp(-\beta \cancel{\varepsilon_{sol}L} + \beta\varepsilon_b + \cancel{L\varepsilon_{sol}}\beta - \beta\varepsilon_{sol}) + 1}$$

$$= \frac{1}{\frac{\Omega}{L} \exp(+\beta(\varepsilon_b - \varepsilon_{sol})) + 1}$$

$$= \frac{1}{\frac{\Omega}{L} e^{+\beta(\varepsilon_b - \varepsilon_{sol})} + 1}$$ (yielding Hill fcn)

d) when ligands are distinguishable, simplified expression is the <u>same</u>

d) cont'd (distinguishable ligands)

STATE	MULTIPLICITY	ENERGY
unbound	$\frac{L \cdot \Omega!}{(\Omega-L)!}$	$L\varepsilon_{sol}$
bound	$\frac{\Omega!}{(\Omega-L+1)!}$	$\varepsilon_b + (L-1)\varepsilon_{sol}$

$$P_{bound} = \frac{\frac{\Omega!}{(\Omega-L+1)!} \exp\left[\beta(\varepsilon_b + (L-1)\varepsilon_{sol})\right]}{\frac{\Omega!}{(\Omega-L)!} \exp(-\beta L \varepsilon_{sol}) + \frac{\Omega!}{(\Omega-L-1)!} \exp\left(-\beta(\varepsilon_b + (L-1)\varepsilon_{sol})\right)}$$

$$= \frac{\Omega^{L-1} \exp\left(-\beta(\varepsilon_b + (L-1)\varepsilon_{sol})\right)}{\Omega^{L} \exp(-\beta L \varepsilon_{sol}) + \Omega^{L-1} \exp\left(-\beta(\varepsilon_b + (L-1)\varepsilon_{sol})\right)}$$

$$= \frac{1}{\Omega \exp\left(-\beta L \cancel{\varepsilon_{sol}} + \beta\varepsilon_b + \beta L \cancel{\varepsilon_{sol}} - \beta\varepsilon_{sol}\right) + 1}$$

$$= \frac{1}{\Omega \exp\left(\beta(\varepsilon_b - \varepsilon_{sol})\right) + 1}$$

... factor of L missing somewhere?

(2)

nucleus $R = 5\,\mu m$

persistence length c

" " of DNA $b = 50\,nm$

base pair size $0.3nm \times 0.3nm \times 2nm$.

a)

2 principle curvatures

$$E_b = K \int dA\, J^2$$ J is curvature.

curvature of sphere $= \frac{1}{R} + \frac{1}{R} = \frac{2}{R}$

bending modulus.

$$= K \int dA\, J^2 = K \int \left(\frac{2}{R}\right)^2 dA = K\left(\frac{2}{R}\right)^2 \int dA$$

$$= K\left(\frac{2}{\cancel{R}}\right)^2 \cdot 4\pi \cancel{R^2}$$

$$= 16K\pi$$

independent of Radius

b) $$\langle R_g \rangle^2 = \frac{Nb^2}{6}$$ (b is persistence length and N is # of steps)

$N =$ 6 billion base pairs. (a diff. N from what is shown above)!

billion $= 10^9$

$$\frac{\text{Tot Length}}{\text{pers. Length}} = \frac{6 \times 10^{+9} \times 0.3\,n\cancel{m}}{50\,n\cancel{m}} = 0.036 \times 10^{+9} \text{ steps.}$$

$$\langle R_g \rangle^2 \sim \frac{0.036 \times 10^{+9} \cdot (50\,nm)^2}{6} \sim 15 \times 10^9$$

$\langle R_g \rangle \sim 122\,000\,nm$

(c)

volume

$$\frac{\text{DNA}_{\text{vol}}}{\text{nucleus}_{\text{vol}}} \sim \frac{6\times10^9 \times 0.3\times0.3\,\text{nm} \times 2\,\text{nm}}{(5\,\mu m)^3}$$

volume of single bp: $0.18\,\text{nm}^3$

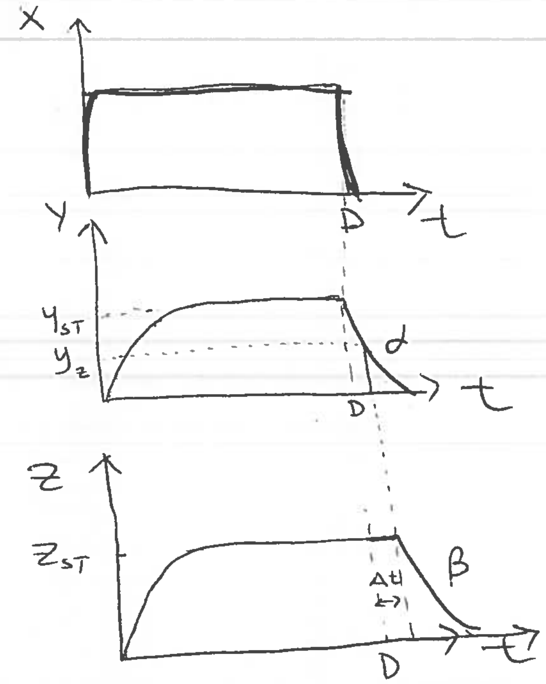
volume of DNA volume: $6\times10^9 \times 0.18\,\text{nm}^3 \sim 10^9\,\text{nm}^3$

Volume of nucleus: $(5\,\mu m)^3 = (5000\,\text{nm})^3 = 125\times10^9\,\text{nm}^3$

$$\frac{10^{9}\,\text{nm}^3}{125\cdot10^{9}\,\text{nm}^3} \sim \boxed{1:125}$$ compaction ratio

(d) $$\frac{R_g^3}{R_{nuc.}^3} \sim \frac{(122\,\mu m)^3}{5^3} \sim \frac{100\cdot100\cdot100}{5\cdot5\cdot5} \sim \boxed{20^3}$$

(3)



y_z is min level of y required to produce z

α & β decay rates for y & z

a)

x	y	z
0	1	1
1	0	1
1	1	1
0	0	0

X OR Y is required to produce Z. Once x is activated, production of both y and z start

b) X reverts rapidly ~~mean~~ (at t=D x=0), y begins decay at t=D. z only begins decaying once y falls below y_z (min level of y required to produce z)

c) $t_{z1/2} = \Delta t + t_{1/2}$

Δt = time for y production to fall from y_{ST} to y_z

$t_{1/2}$ = time for z to drop from Z_{ST} to $\frac{Z_{ST}}{2}$

Δt:

$$y_z = y_{ST} e^{-\alpha \Delta t} \qquad \frac{y_z}{y_{ST}} = e^{-\alpha \Delta t} \qquad \ln\left(\frac{y_z}{y_{ST}}\right) = -\alpha \Delta t$$

$$\ln y_z - \ln y_{ST} = -\alpha \Delta t$$

$$\Delta t = -\frac{1}{\alpha}(\ln y_z - \ln y_{ST})$$

$$= \frac{1}{\alpha} \ln\left(\frac{y_{ST}}{y_z}\right)$$

$t_{1/2}$

$$\frac{1}{2} = e^{-\beta t_{1/2}}$$

$$t_{1/2} = \frac{\ln 2}{\beta}$$

$$\boxed{t_{z1/2} = \frac{\ln 2}{\beta} + \frac{1}{\alpha} \ln\left(\frac{y_{ST}}{y_z}\right)}$$

~~wrt D~~ wrt t=D

(4) Reading.

a) lipid ordered versus disordered
DOPC prefers disordered phase (L_d)
Sphingomyelin & cholesterol enrich in liquid phase w/ short-range order (L_o)

b) Fig 1 a & b show similar shapes even though domains are reversed, suggests that bending resistance diff. (or curvature preferences) are not dominant in determining global ves. shape.
... Line tension between coexisting domains has been proposed to control membrane deformation

(5) Reading.

a) Excluded volumes of each sphere intersect; allows for more volume available to smaller spheres (reduced excluded volume); entropic gain results in min of free energy.

b) shorter strands (fig 3a)
since less attractive energy is required to form a loop.